

Simple tools for extrapolations of human mortality in rich countries

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Abstract: Suitable assumptions for the Gompertz mortality law take into account the break in the time development observed recently by Wilmoth et al. They show how a drastic reduction in the birth rate and improved living conditions lead to a drastic increase in the fraction of old people in the population, and how immigration of half a percent of the population per year can mostly stop this increase.

Human life expectancies increased and birth rates decreased, in the 20th century in most rich countries, and this may happen soon elsewhere [1]. Predictions for the future [2-4] are usually extrapolations of the trends of the past and thus are problematic if extended over centuries as in ref.4. Recent analysis of the age of the oldest people in a finite population versus time [5], of survival probabilities as a function of life expectancy at birth [6], or of survival probabilities as a function of survival probability up to age 40 [7] raise the possibility of sharp breaks in these curves. By definition, no extrapolation beyond the next (unknown) break is possible, if these breaks are infinitely sharp (if infinitely accurate data would be available). Now we try to describe these effects by a simple model with a limited number of parameters, and show the versatility of this model by an extrapolation of the fraction of old people under various hypotheses. While some aspects are motivated by German data, the one-page computer program could easily be adapted to other rich countries with birth rates below replacement level and is available from stauffer@thp.uni-koeln.de.

Human adult mortalities between the ages x of 30 and 90 years follow roughly an exponential increase $\propto \exp(bx)$, known as the Gompertz law since the 19th century. Much of the trends of the past centuries in various countries were summarized by the universality law [8]:

$$\mu/b = A e^{b(x-X)}$$

where μ is the mortality function, A and $X \simeq 103$ years are universal over the whole human species, and only the slope b increases with increasing progress. The time unit is one year throughout. (Also Thatcher [9] noted that human mortality at about 100 years is roughly constant.) We take b to increase linearly with time from 0.07 in 1821 to 0.093 in 1971, and from then on to remain constant. Fig.1a compares the mortality function of German women with this simple assumption. We see that our analysis is valid only at older ages and only for the last decades of the 20th century in rich countries when child mortality was already low. Fig.1b compares the life expectancy at age 65 for German men with simulations of the model explained below.

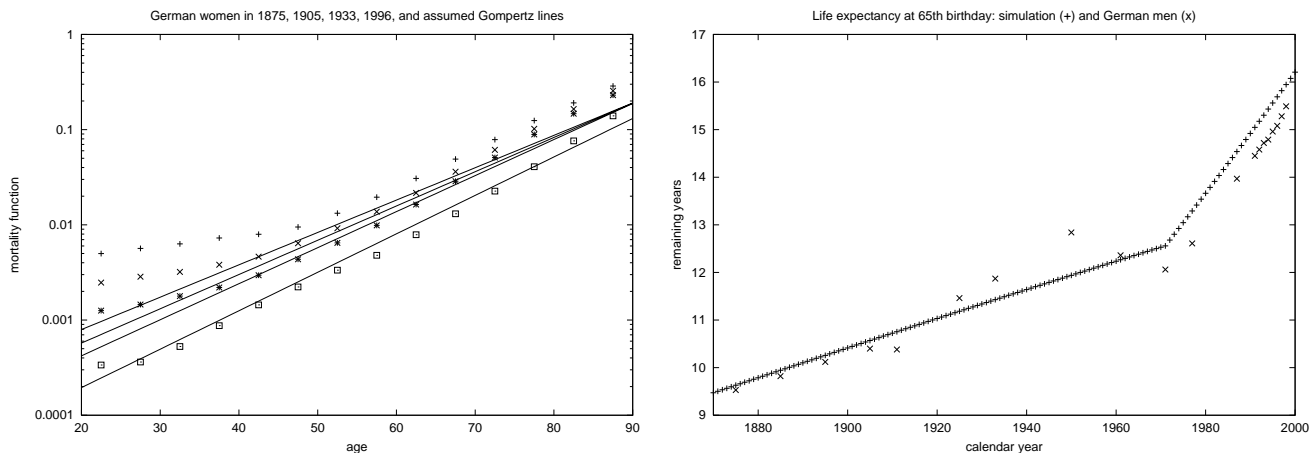


Figure 1: a) German mortality functions and Gompertz law, for 1875, 1905, 1933, and 1996. b) Remaining life expectancy for German men at age 65, and simulation results from the simple model presented later.

(In the above equation, the mortality function $\mu = -d \ln S(x)/dx$ is also called the hazard factor or force of mortality [10] and is approximated by $\ln[S(x-1/2)/S(x+1/2)]$ where $S(x)$ (often denoted as $l(x)$) is the probability to survive from birth to age x . For centenarians, downward deviations of μ from the Gompertz law have been reported [10] but an extrapolation to zero systematic errors (from age overstating) seems lacking.)

However, these assumptions are still wrong since they lead for increasing b (increasing progress) to a *decrease* of the average maximum age in a finite population. (This maximum age x_{max} is defined as the age at which the reciprocal survival probability $1/S(x_{max})$ reaches the population size [11]; we ignore the growth of the population with time and also the variation of the mortality parameters during the lifetime of an individual.) In reality, x_{max} increased with increasing progress during the last century [5], page 46 in [1].

To allow for x_{max} to increase with calendar year t , we allow the characteristic age X to increase with time after 1971, by generalizing $X = 103$ to $X = 103 + 0.15(t - 1971)$. This generalization is already included in Fig.1 and slightly separates the lowest straight line from the three higher (earlier) ones. Now x_{max} increases with increasing t , similar to reality (page 46 in [1].)

The two effects, the increase of b before the year 1971, and the increase of x thereafter, describe the two empirical observations reviewed recently by Yashin et al [12]: During the first half the the 20th century the survival curves $S(x)$ became more rectangular (“compression of mortality”), but later this rectangularization stopped and is replaced by a shifting of the old-age survival curve towards older ages at constant shape. The interquartile death rate is the number of years between

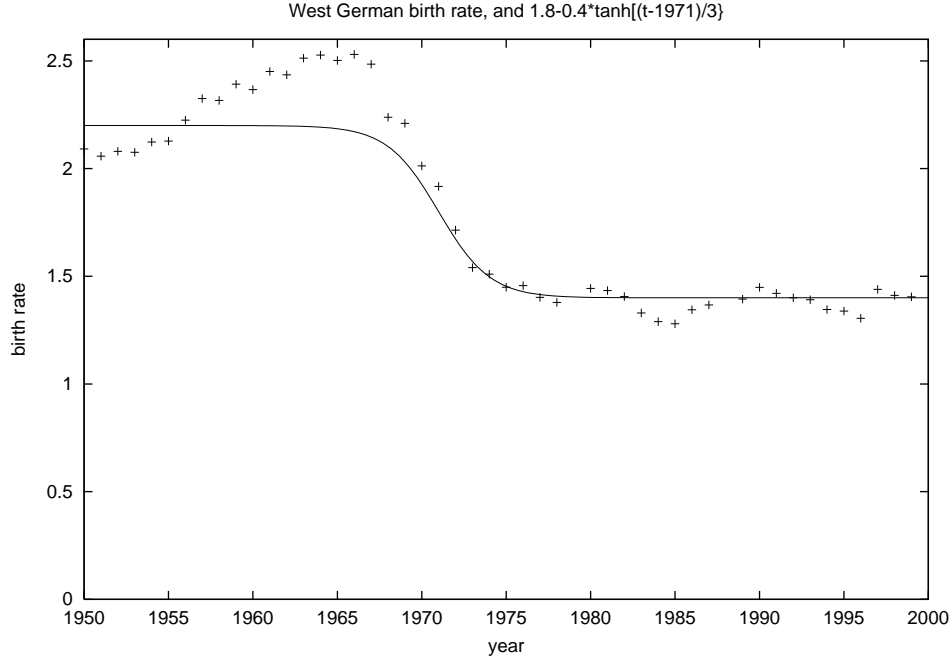


Figure 2: West German birth rate in recent decades, and fitted tanh formula.

$S(x) = 0.75$ and $S(x) = 0.25$. In my calculations it first declines and then stays constant similar to Swedish reality (Fig.4a of Wilmoth and Horiuchi [5], if deaths before age 30 are ignored).

For the birth rate we assume that pregnancies happen at ages 20 to 40 only, with equal probability in each of these 20 years. We first adjust a birth rate $m = 2.17$ such that for $b = 0.07$ (a realistic value for earlier centuries [9]) a stationary population is reached. (Note that “births” here count only babies reaching adult age; child mortality was ignored here from the beginning.) The birth rate then increased linearly in time to 2.2 for the year 1971. Then very rapidly it decreases to 1.4 and stays there, according to a hyperbolic tangent formula

$$m(t) = 1.8 - 0.4 \cdot \tanh[(t - 1971)/3]$$

compared in Fig.2 with West German reality.

Thus, compatible with Wilmoth et al [5], the year 1971 is taken as both the year when rectangularization was replaced by shifting in $S(x)$, and then the birth rate had its strongest decline. These sharp changes in the higher time derivatives may be missing in traditional demographic extrapolations [2-4] but could be easily implemented in a Fortran program of about 50 lines. Thus these methods could be used in more realistic fits and extrapolations for specific countries.

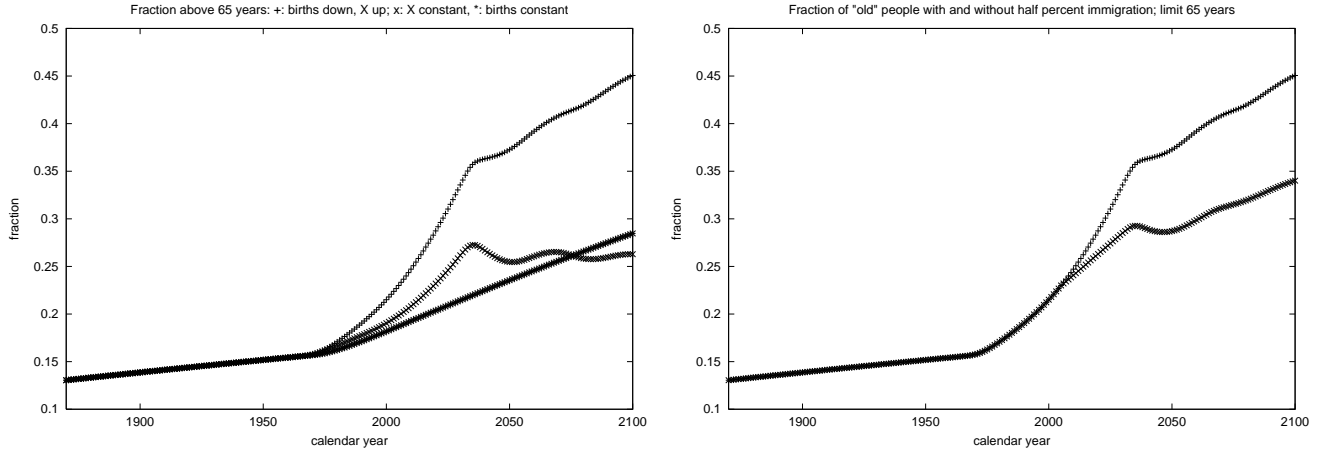


Figure 3: a) Fraction of old people in the computer simulation (x), together with the hypothetical development had the birth rate not dropped (+) or had the characteristic age X not increased (*). b) Same higher curve from part a, compared with lower curve assuming after 2005 immigration to stabilize population.

Figure 3 illustrates from these assumptions the future development for the fraction of “old” people, the limit taken at 65. About this “age quake” much has been talked, not done. In Germany, for example, three quarters of the population prefer to retire before the age of 60, and a new immigration law is not even called by that name. Figure 3a also shows what would have happened if the birth rate would not have dropped, or if the characteristic age X would not increase. Then, the increase in the fraction of old people would be much weaker. More realistically, Fig.3b shows the changes made by an immigration of people aged 6 to 40, amounting to half a percent of the total population per year and roughly stabilizing the population.

In summary, the possible breaks in the time development of demographic parameters were included in a simple approximation which could be incorporated into more detailed traditional models of human population dynamics. Simulations are in preparation [13] to check whether these phenomenological assumptions can be reproduced by more microscopic models [14].

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